Bonus Lesson 1 - Arc Length

If f is continuous on $[a,b]$ and differentiable on (a,b), then the arc length of the graph of f from $(a,f(a))$ to $(b,f(b))$ is: $\int_a^b \sqrt{1+(f'(x))^2}\,dx$

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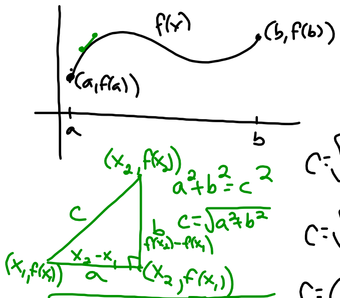

$a^2+b^2=c^2$

$c=\sqrt{a^2+b^2}$

$c=\sqrt{(x_2-x_1)^2+(f(x_2)-f(x_1))^2}$

$c=\sqrt{(x_2-x_1)^2\left(1+\frac{(f(x_2)-f(x_1))^2}{(x_2-x_1)^2}\right)}$

$c=\sqrt{(x_2-x_1)^2\left(1+(f'(x))^2\right)}$

$c=(x_2-x_1)\sqrt{(1+(f'(x))^2}$

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Ex. Find the length of $f(x)=x^{3/2}$ $[1,9]$

$f'(x)=\frac{3}{2}x^{1/2}$

$(f'(x))^2=\frac{9}{4}x$

$\int_1^9 \sqrt{1+\frac{9}{4}x}\,dx$

$u=1+\frac{9}{4}x$

$du=\frac{9}{4}dx$

$\frac{4}{9}\int \sqrt{u}\,du = \frac{4}{9}\cdot\frac{2}{3}\left(1+\frac{9}{4}x\right)^{3/2}\Big|_1^9$

$\frac{8}{27}\left(1+\frac{81}{4}\right)^{3/2}-\frac{8}{27}\left(1+\frac{9}{4}\right)^{3/2}$

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1. $y=\frac{1}{3}(x^2+2)^{3/2}$

$y'=\frac{1}{\cancel{3}}(2x)\frac{\cancel{3}}{2}(x^2+2)^{1/2}$

$(y')^2=x^2(x^2+2)$

$\int_0^3 \sqrt{1+x^4+2x^2}\,dx$

$\int_0^3 \sqrt{x^4+2x^2+1}\,dx$

$\int_0^3 \sqrt{(x^2+1)^2}\,dx$

$\int_0^3 x^2+1\,dx$

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3. $9x^2=4y^3$

$x^2=\frac{4}{9}y^3$

$x=\frac{2}{3}y^{3/2}$

$x'=y^{1/2}$

$(x')^2=y$

$\int_0^3 \sqrt{1+y}\,dy$

$u=1+y$

$du=dy$

$\int u^{1/2}\,du$

$\frac{2}{3}(1+y)^{3/2}\Big|_0^3$

$\frac{2}{3}(4)^{3/2}-\frac{2}{3}$

$\frac{16}{3}-\frac{2}{3}$

$\frac{14}{3}$

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5. $y=\frac{1}{3}x^3+\frac{1}{4x}$ $\frac{1}{4}x^{-1}$

$y'=x^2-\frac{1}{4}x^{-2}$

$(y')^2=(x^2-\frac{1}{4}x^{-2})^2$

$\int_1^3 \sqrt{1+x^4-\frac{1}{2}+\frac{1}{16}x^{-4}}\,dx$

$\int_1^3 \sqrt{x^4+\frac{1}{2}+\frac{1}{16}x^{-4}}\,dx$

$\int_1^3 \sqrt{(x^2+\frac{1}{4}x^{-2})^2}\,dx$

$\int_1^3 x^2+\frac{1}{4}x^{-2}\,dx$

$\boxed{\frac{53}{6}}$

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