

Bonus lesson 1! Integration by Trig Substitution

I. Review

List all 3 Pythagorean Identities

- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \csc^2\theta$

Double L
 $\sin 2\theta = 2\sin\theta\cos\theta$

List Both Power Reduction Identities

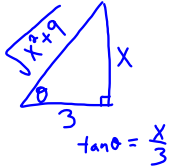
- $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$
- $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

May 18-7:18 AM

II Examples

1. $\int \frac{dx}{\sqrt{9+x^2}}$

$x = 3\tan\theta$
 $dx = 3\sec^2\theta d\theta$



$\int \frac{3\sec^2\theta d\theta}{\sqrt{9+9\tan^2\theta}}$

$\int \frac{3\sec^2\theta d\theta}{3\sqrt{1+\tan^2\theta}}$

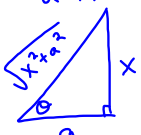
$\int \sec\theta d\theta = \ln|\sec\theta + \tan\theta| + C$

$\ln\left|\frac{\sqrt{x^2+9}}{3} + \frac{x}{3}\right| + C$

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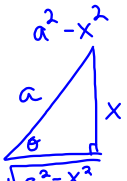
Scenarios

$\frac{dx}{\sqrt{a^2+x^2}}$



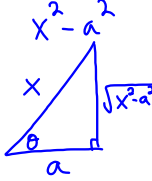
Use $x = a\tan\theta$

$\frac{dx}{\sqrt{a^2-x^2}}$



Use $x = a\sin\theta$

$\frac{dx}{\sqrt{x^2-a^2}}$

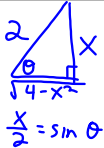


Use $x = a\sec\theta$

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2. $\int \frac{\sqrt{4-x^2}}{x^2} dx$

$x = 2\sin\theta$
 $dx = 2\cos\theta d\theta$



$\int \frac{\sqrt{4-4\sin^2\theta} \cdot 2\cos\theta d\theta}{4\sin^2\theta}$

$\int \frac{2\cos\theta \cdot 2\cos\theta}{4\sin^2\theta} d\theta = \int \cot^2\theta d\theta$

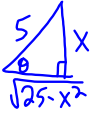
$\int \csc^2\theta - 1 d\theta = -\cot\theta - \theta + C$

$-\frac{\sqrt{4-x^2}}{x} - \arcsin\frac{x}{2} + C$

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3. $\int \frac{dx}{\sqrt{25-x^2}}$

$x = 5\sin\theta$
 $dx = 5\cos\theta d\theta$



$\int \frac{5\cos\theta d\theta}{\sqrt{25-25\sin^2\theta}}$

$\int \frac{5\cos\theta d\theta}{5\sqrt{1-\sin^2\theta}}$

$\int d\theta$

$\theta + C = \arcsin\frac{x}{5} + C$

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